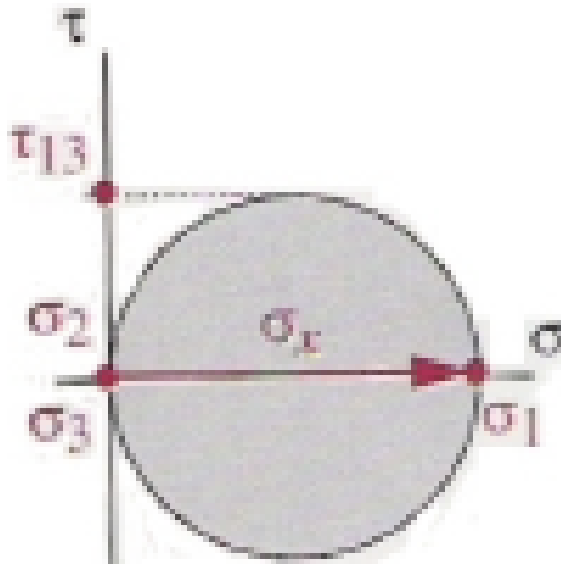


Failure Theories

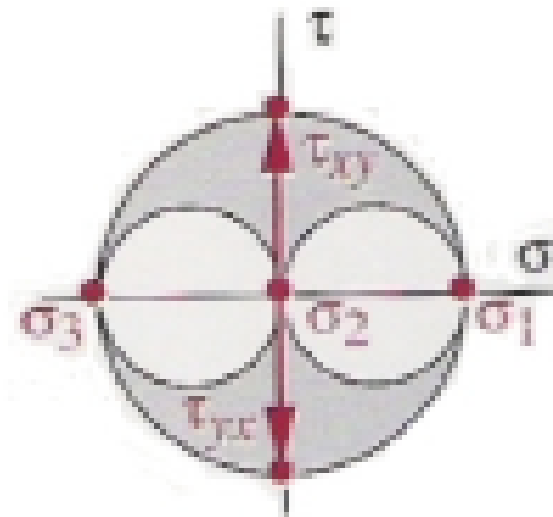
Why do mechanical components fail? Mechanical components fail because the applied stresses exceeds the material's strength (Too simple).

What kind of stresses cause failure? Under any load combination, there is always a combination of normal and shearing stresses in the material.

Pure Tension



Pure Shear



What is the definition of Failure?

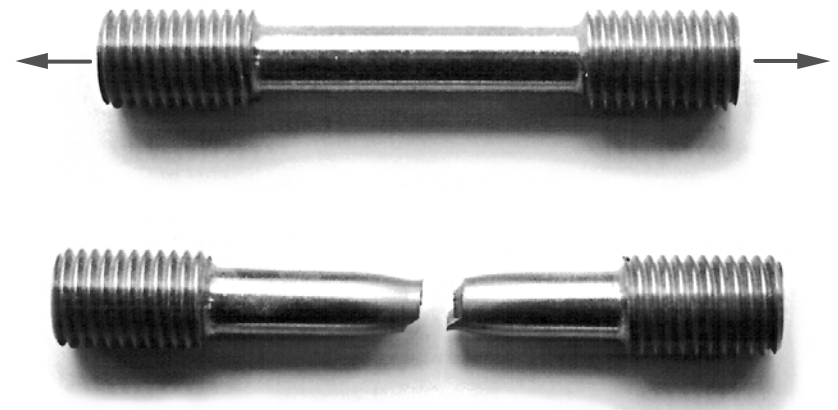
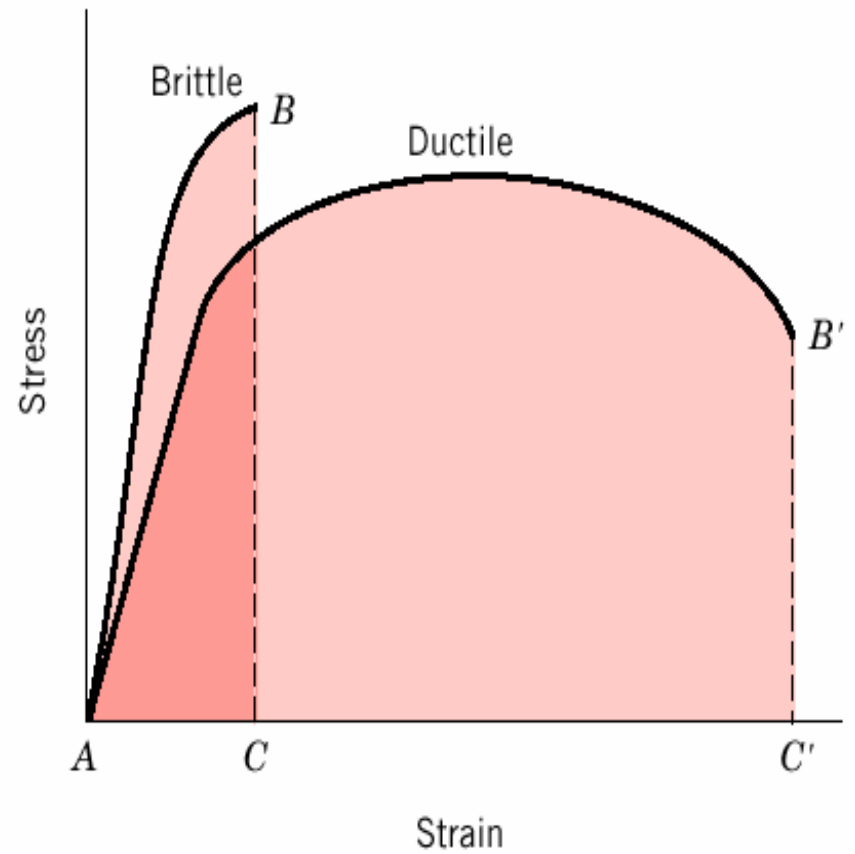
Obviously fracture but in some components yielding can also be considered as failure, if yielding distorts the material in such a way that it no longer functions properly

Which stress causes the material to fail?

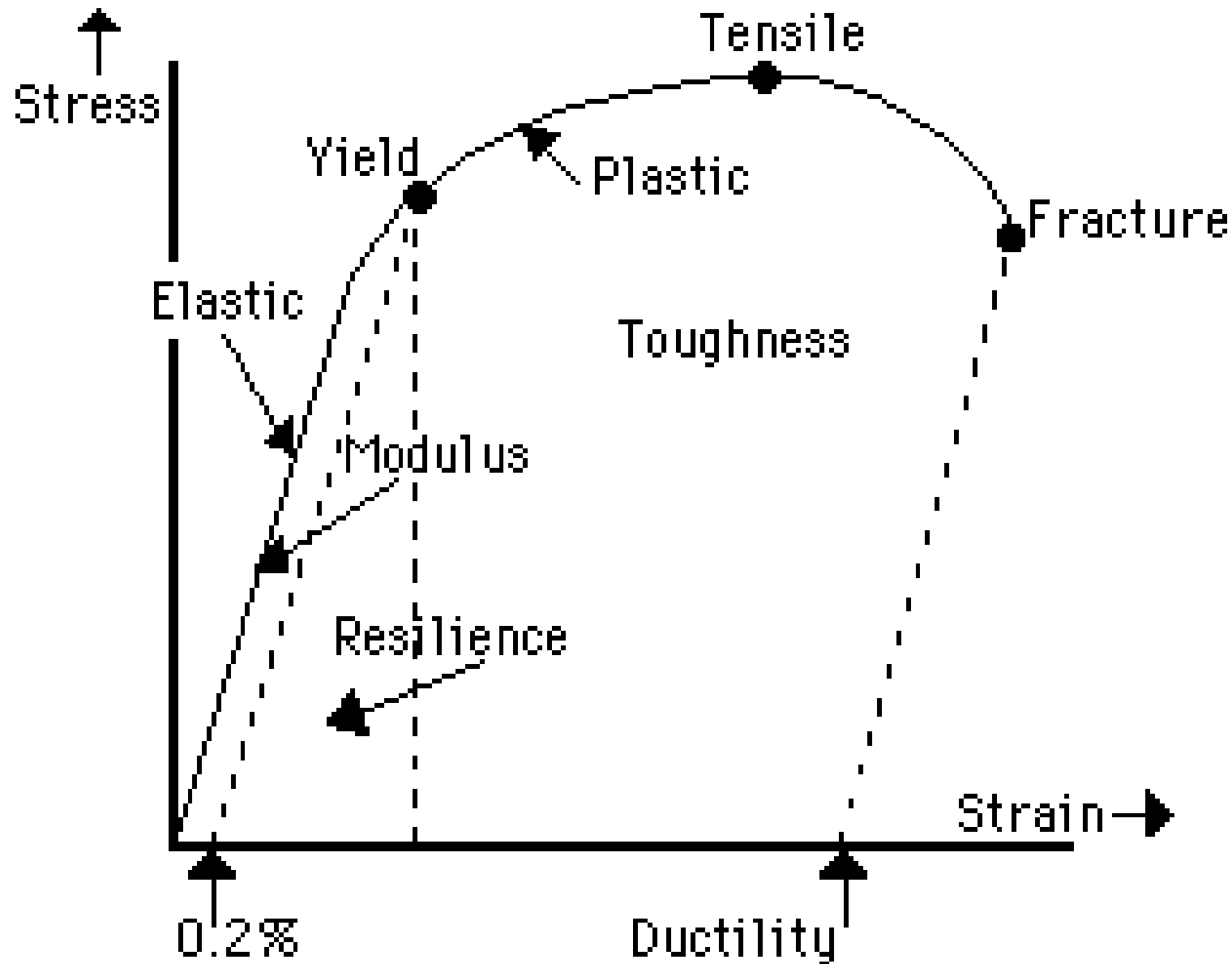
Usually ductile materials are limited by their shear strengths.

While brittle materials (ductility < 5%) are limited by their tensile strengths.

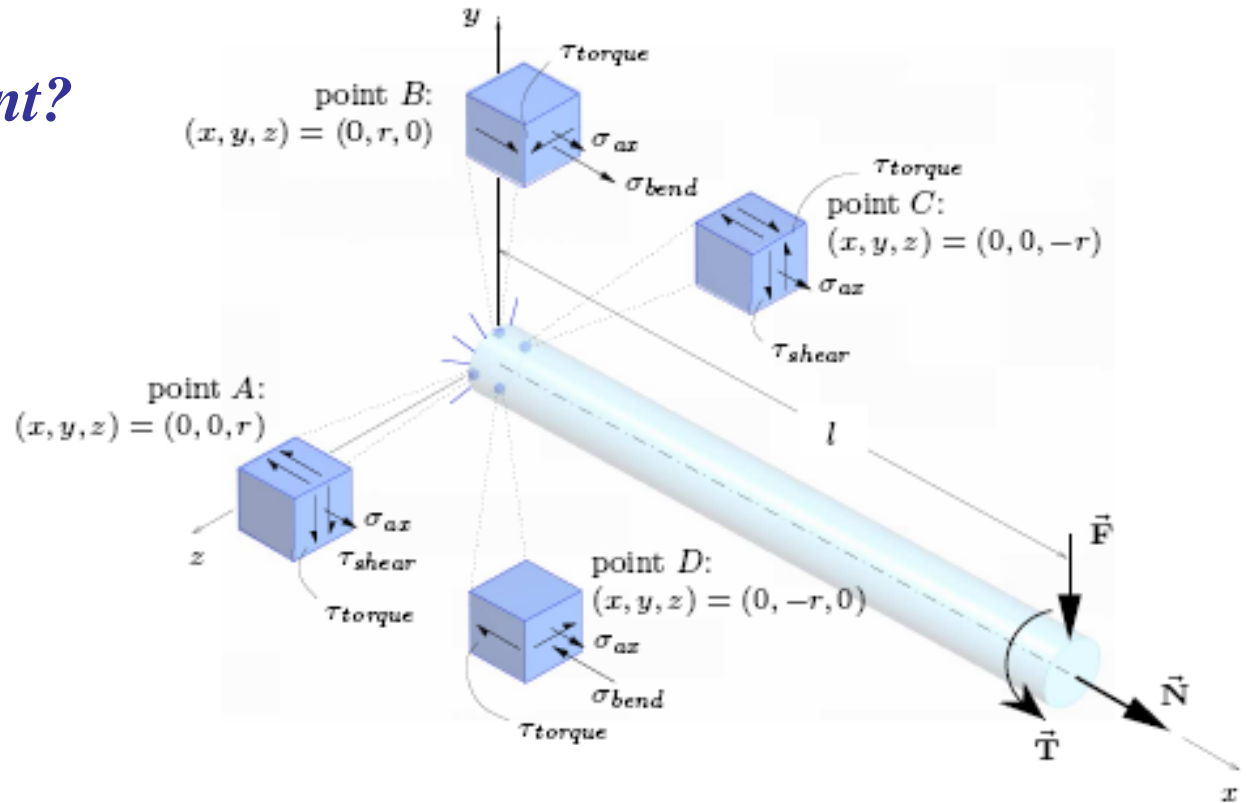
Stress at which point?



Stress-Strain Curve



Stress at which point?



<p>point A:</p> $\sigma_x = \sigma_{ax} = \frac{N}{A}$ $\tau_{xy} = -\tau_{torque} - \tau_{shear} = -\frac{T_r}{J} - \frac{4F}{3A}$ $\sigma_y = \sigma_z = \tau_{xz} = \tau_{yz} = 0$	<p>point B:</p> $\sigma_x = \sigma_{ax} + \sigma_{bend} = \frac{N}{A} + \frac{Flr}{I_z}$ $\tau_{xz} = \tau_{torque} = \frac{T_r}{J}$ $\sigma_y = \sigma_z = \tau_{xy} = \tau_{yz} = 0$
<p>point C:</p> $\sigma_x = \sigma_{ax} = \frac{N}{A}$ $\tau_{xy} = \tau_{torque} - \tau_{shear} = \frac{T_r}{J} - \frac{4F}{3A}$ $\sigma_y = \sigma_z = \tau_{xz} = \tau_{yz} = 0$	<p>point D:</p> $\sigma_x = \sigma_{ax} - \sigma_{bend} = \frac{N}{A} - \frac{Flr}{I_z}$ $\tau_{xz} = -\tau_{torque} = -\frac{T_r}{J}$ $\sigma_y = \sigma_z = \tau_{xy} = \tau_{yz} = 0$

Failure Theories

Load type
Uniaxial
Biaxial
Pure Shear

Material Property
Ductile
Brittle

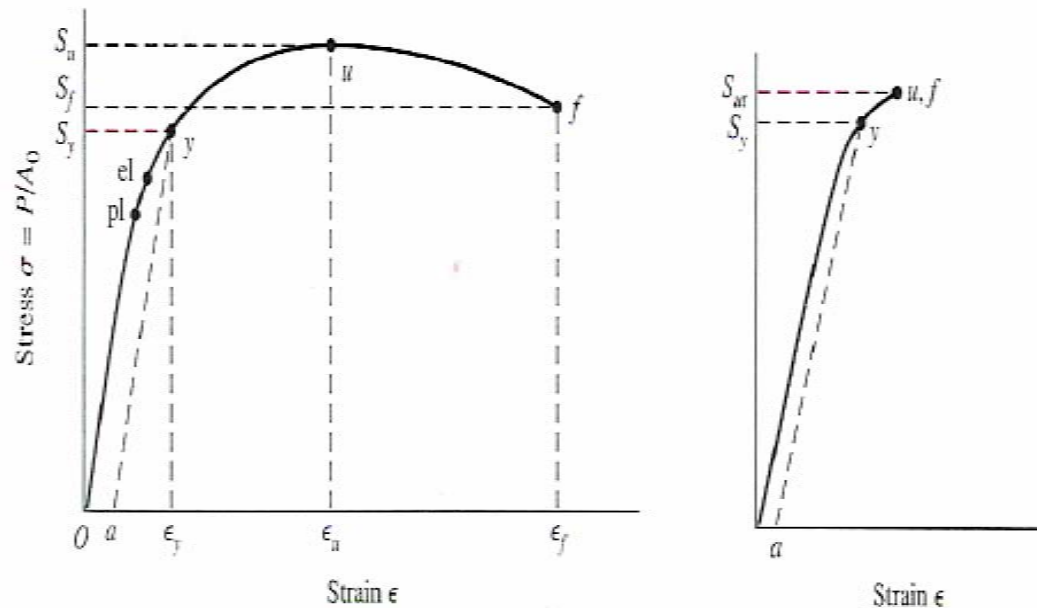
Application of Stress
Static
Dynamic

Static Loading
Maximum Normal Stress
Modified Mohr
Yield strength
Maximum shear stress
Distortion energy

Dynamic Loading
Goodman
Gerber
Soderberg

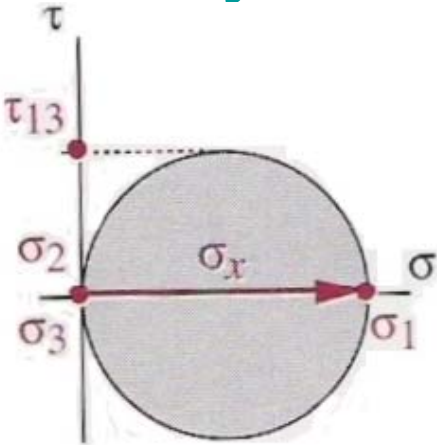
Static Failure Theories

The idea behind the various classical failure theories is that *whatever is responsible for failure in the standard tensile test will also be responsible for failure under all other conditions of static loading.*



	Ductile Material	Brittle Material
Characteristic Failure Stress	Yield Stress	Ultimate Stress
Important Theories	<ol style="list-style-type: none"> 1. Maximum Shear Stress 2. Maximum Octahedral Shear Stress 	<ol style="list-style-type: none"> 1. Maximum Normal Stress 2. Modified Mohr.

Maximum Shear Stress Theory



Ductile Materials

Failure occurs when the maximum shear stress in the part exceeds the shear stress in a tensile test specimen (of the same material) at yield.

Hence in a tensile test,

$$\tau_{\max} = \frac{S_y}{2}$$

For a general state of stresses

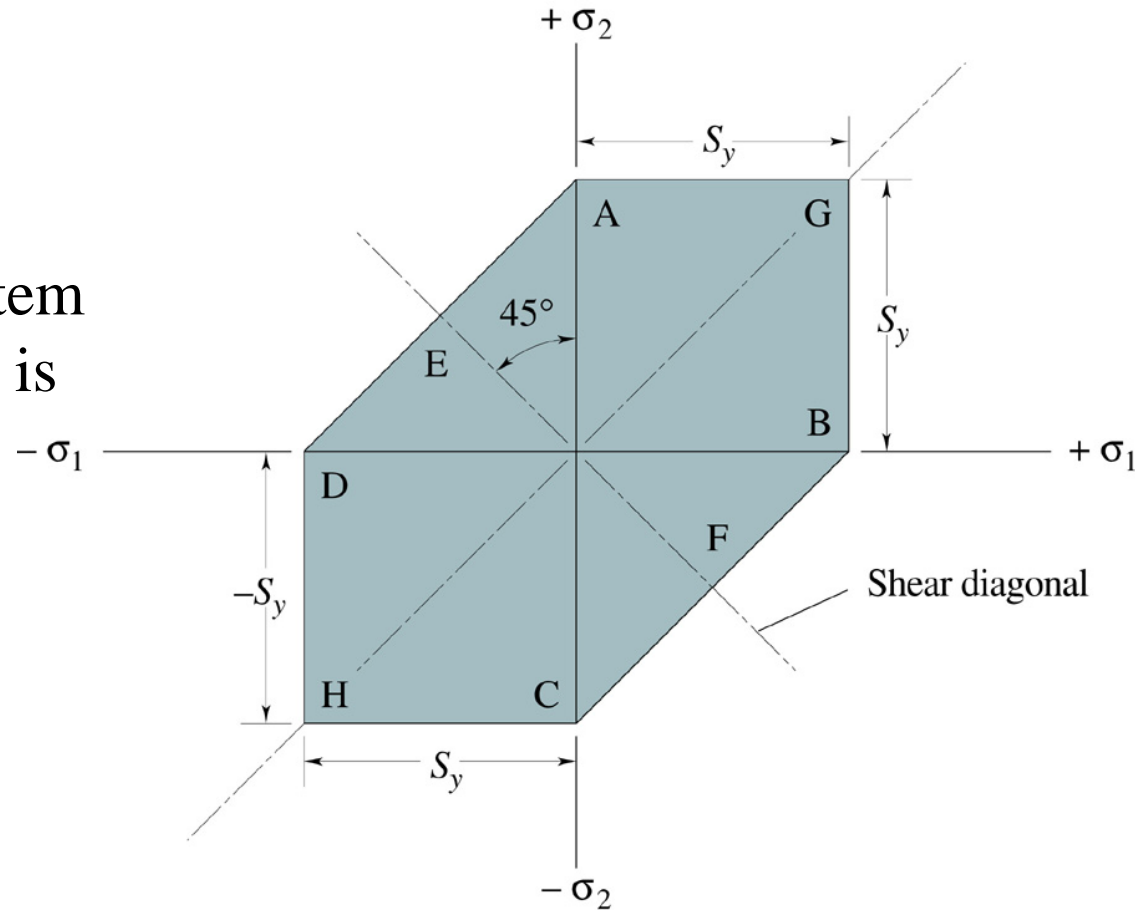
$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{S_y}{2}$$

This leads to an hexagonal failure envelop. A stress system in the interior of the envelop is considered **SAFE**

The Maximum Shear Stress Theory for Ductile Materials is also known as the *Tresca Theory*.

for design purposes, the failure relation can be modified to include a factor of safety (n):

$$n = \frac{S_y}{\sigma_1 - \sigma_3}$$



Several cases can be analyzed in plane stress problems:

Case 1: $\sigma_1 \geq \sigma_2 \geq 0$

In this case $\sigma_3 = 0$

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_1}{2} = \frac{S_y}{2}$$

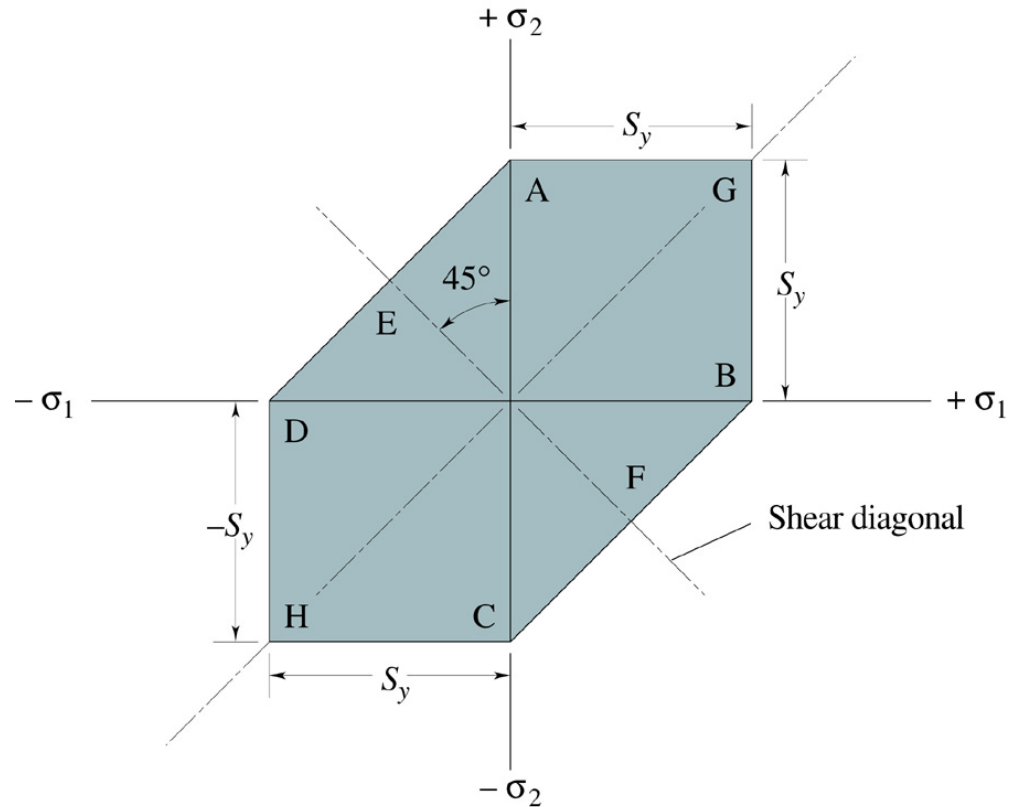
$$\sigma_1 \geq S_y$$

Case 2: $\sigma_1 \geq 0 \geq \sigma_3$

Yielding condition

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{S_y}{2}$$

$$\sigma_1 - \sigma_3 \geq S_y$$



Distortion Energy Theory

Based on the consideration of angular distortion of stressed elements.

The theory states that failure occurs when the distortion strain energy in the material exceeds the distortion strain energy in a tensile test specimen (of the same material) at yield.

Resilience

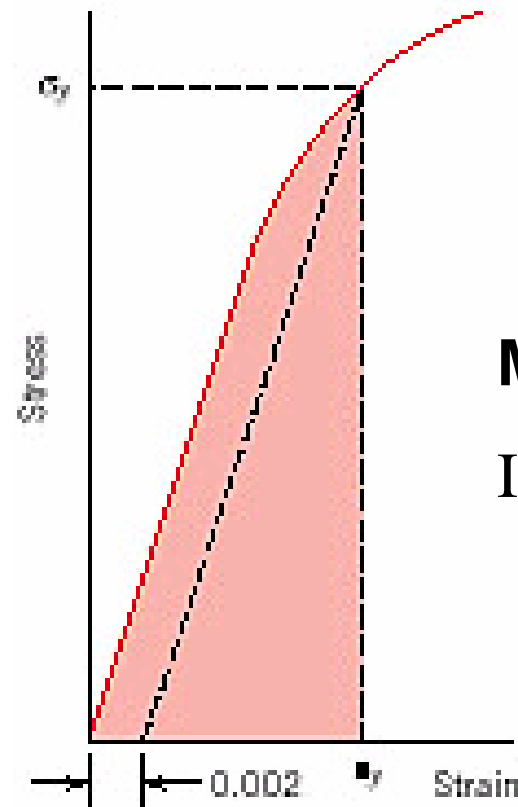
Resilience is the capacity of a material to absorb energy when it is deformed elastically and then, upon unloading, to have this energy recovered.

Modulus of resilience U_r

$$U_r = \int_0^{\epsilon_y} \sigma d\epsilon$$

If it is in a linear elastic region,

$$U_r = \frac{1}{2} \sigma_y \epsilon_y = \frac{1}{2} \sigma_y \left(\frac{\sigma_y}{E} \right) = \frac{\sigma_y^2}{2E}$$



For general 3-D stresses: $u = \frac{1}{2}(\sigma_1 \varepsilon_1 + \sigma_2 \varepsilon_2 + \sigma_3 \varepsilon_3)$

Applying Hooke's Law $u = \frac{1}{2E}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1))$

There are two components in this energy a mean component and deviatoric component.

$$\sigma_M = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \frac{\sigma_x + \sigma_y + \sigma_z}{3}$$

$$\sigma_{1,D} = \sigma_1 - \sigma_M \quad \sigma_{2,D} = \sigma_2 - \sigma_M \quad \sigma_{3,D} = \sigma_3 - \sigma_M$$

The energy due to the mean stress (it gives a volumetric change but not a distortion:

$$u_{Mean} = \frac{1}{2E}(\sigma_M^2 + \sigma_M^2 + \sigma_M^2 - 2\nu(\sigma_M \sigma_M + \sigma_M \sigma_M + \sigma_M \sigma_M))$$

$$u_{Mean} = \frac{1}{2E}[3\sigma_M^2(1-2\nu)] = \frac{1-2\nu}{6E}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2\sigma_1\sigma_2 + 2\sigma_2\sigma_3 + 2\sigma_3\sigma_1)$$

$$u_D = u - u_{Mean} = \frac{1+\nu}{3E} (\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1)$$

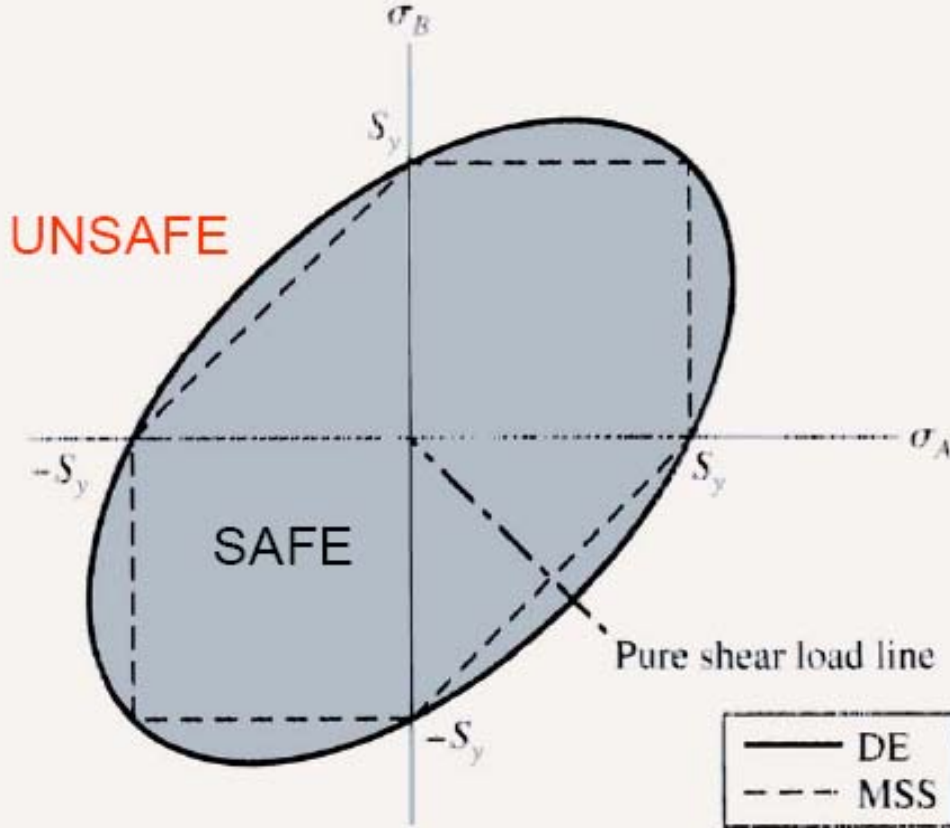
Compare the distortion energy of a tensile test with the distortion energy of the material.

$$u_{Tensile} = \frac{1+\nu}{3E} S_y^2 = u_D = \frac{1+\nu}{3E} (\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1)$$

$$S_y = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1}$$

$$S_y = \sqrt{\sigma_1^2 + \sigma_3^2 - \sigma_3\sigma_1} \quad \text{Plane Stress}$$

Von Mises effective stress : Defined as the uniaxial tensile stress that creates the same distortion energy as any actual combination of applied stresses.



This simplifies the approach since we can use the following failure criterion

$$\sigma_{VM} \geq S_y$$

$$n = \frac{S_y}{\sigma_{VM}}$$

$$\sigma_{VM} = \sqrt{\frac{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}{2}}$$

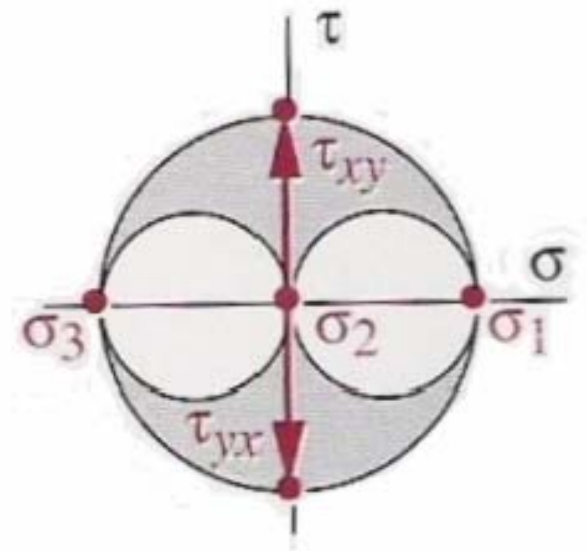
$$\sigma_{VM} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2} \quad 2D$$

Case of Pure Shear

$$\sigma_1 = \tau, \sigma_2 = 0, \sigma_3 = -\tau$$

$$\sigma_{VM} = \sqrt{3}\tau_{xy} \geq S_y$$

$$\tau_{Max} = \frac{S_y}{\sqrt{3}} = 0.577S_y$$



Brittle Materials

Several theories have been developed to describe the failure of brittle materials, such as:

Maximum Normal Stress Theory

Coulomb-Mohr Theory

Modified-Mohr Theory

Maximum Normal Stress Theory

Failure occurs when one of the three principal stresses reaches a permissible strength (TS).

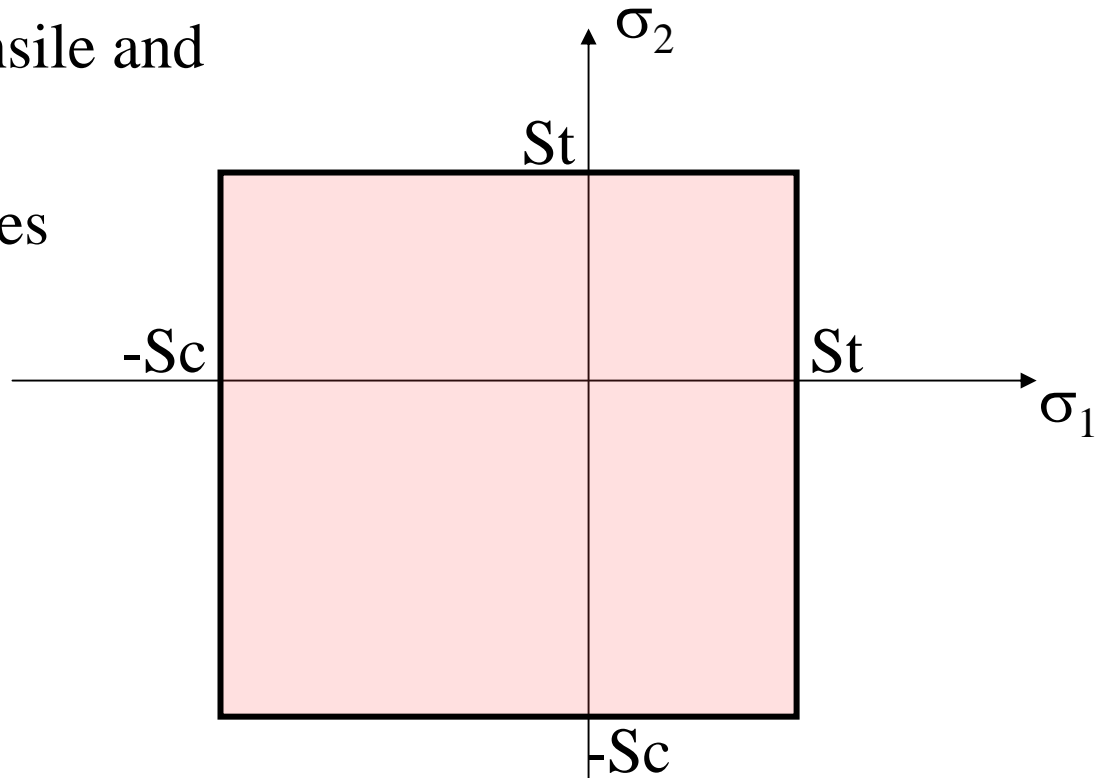
$$\sigma_1 > \sigma_2$$

Failure is predicted to occur when

$$\sigma_1 = S_t \text{ and } \sigma_2 < -S_c$$

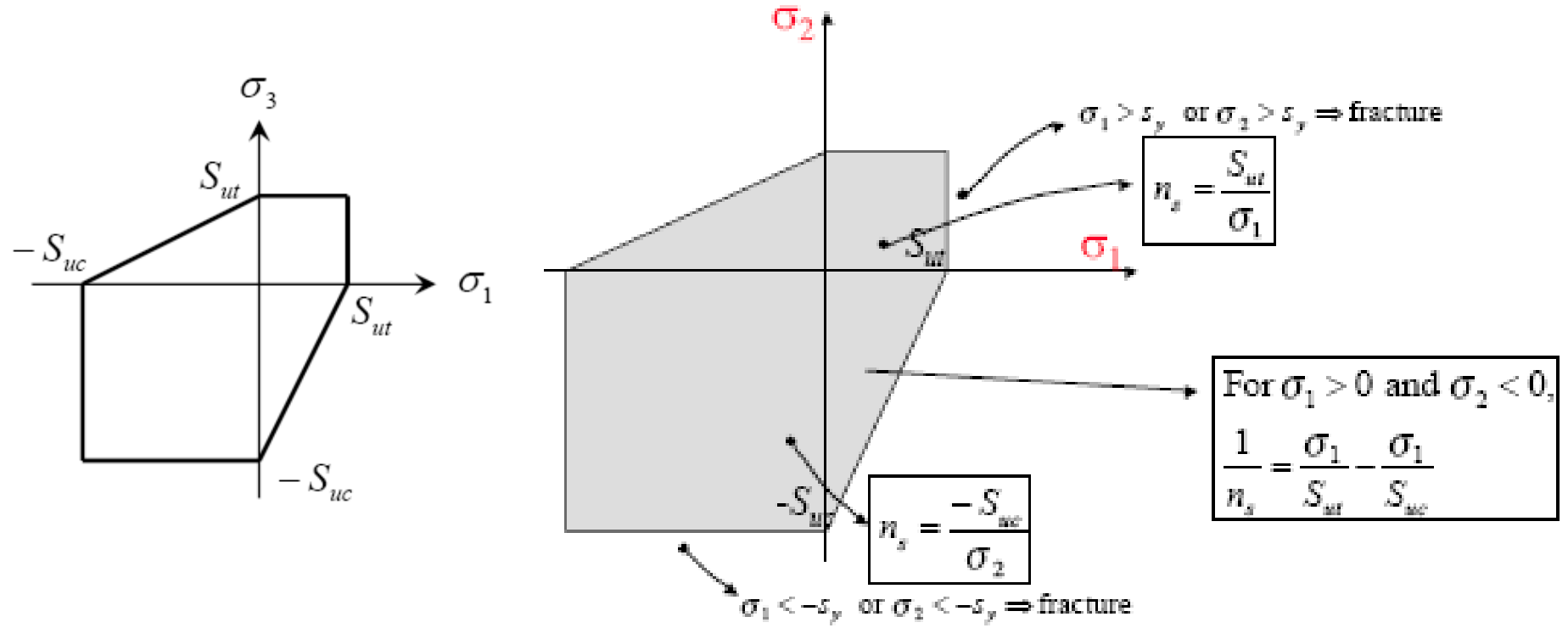
Where S_t and S_c are the tensile and compressive strength

For a biaxial state of stresses



Coulomb-Mohr Theory or Internal Friction Theory (IFT)

This theory is a modification of the maximum normal stress theory in which the failure envelope is constructed by connecting the opposite corners of quadrants I and III.



n_s is whichever is smallest

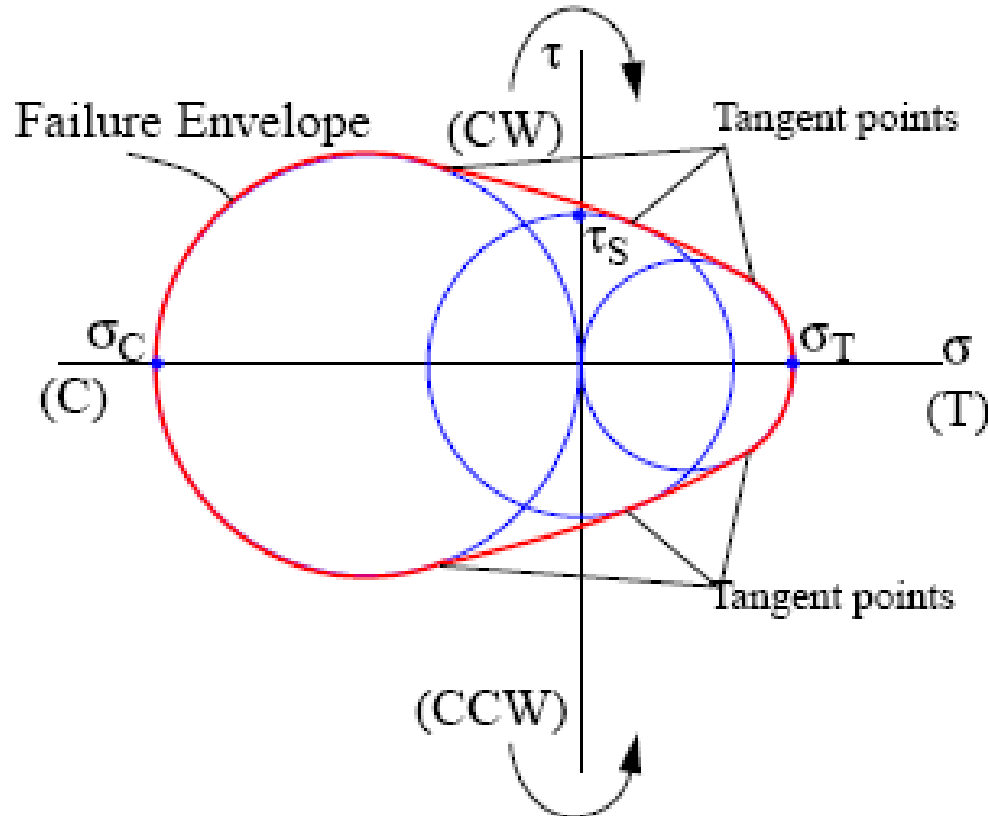
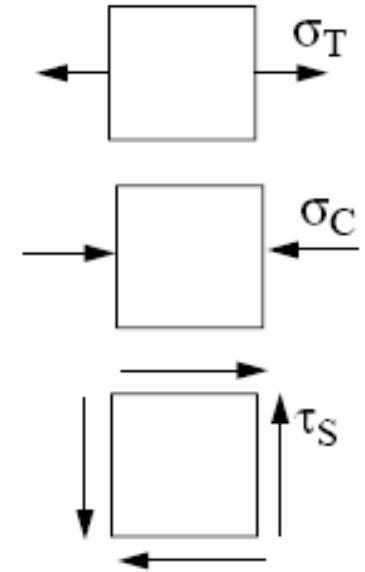
The result is an hexagonal failure envelop.

Similar to the maximum shear stress theory but also accounts for the uneven material properties of brittle material

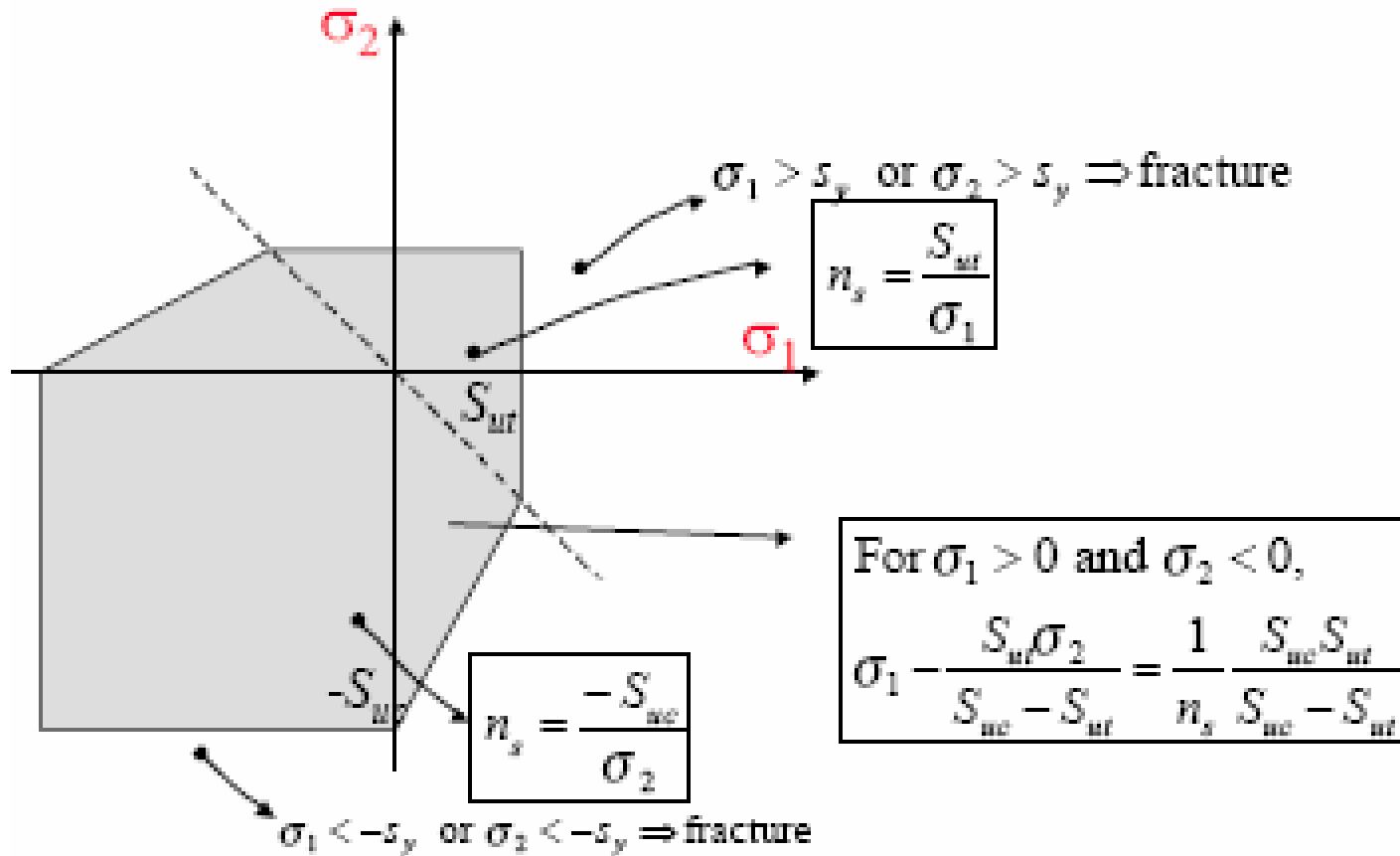
Mohr's Theory

The theory predicts that a material will fail if a stress state is on the envelope that is tangent to the three Mohr's circles corresponding to:

- uni-axial ultimate stress in tension,
- uni-axial ultimate stress in compression, and
- pure shear.

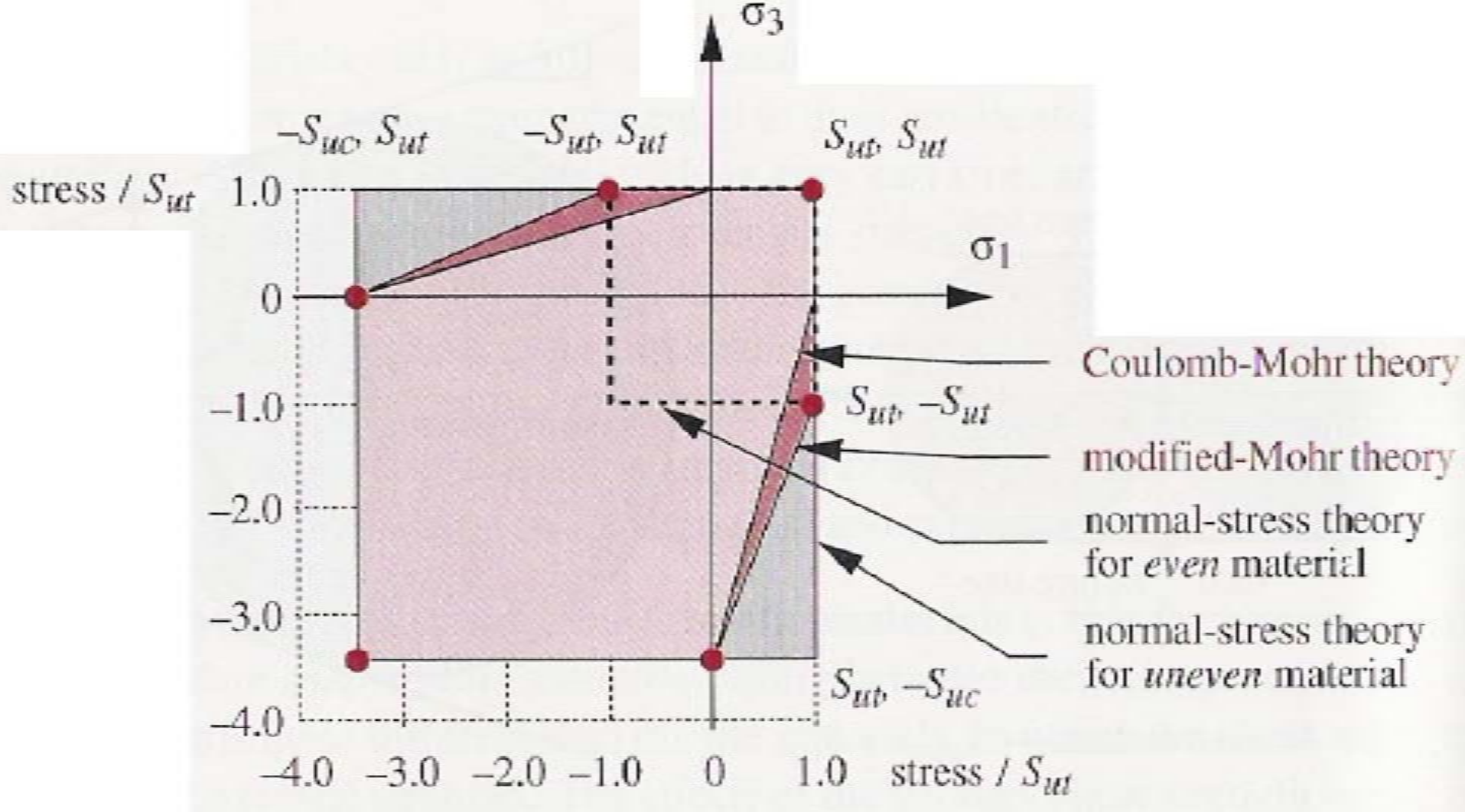


Modified Mohr's Theory



n_s is whichever is smallest

This theory is a modification of the Coulomb-Mohr theory and is the *preferred theory for brittle materials*.



Maximum Normal-Strain Theory

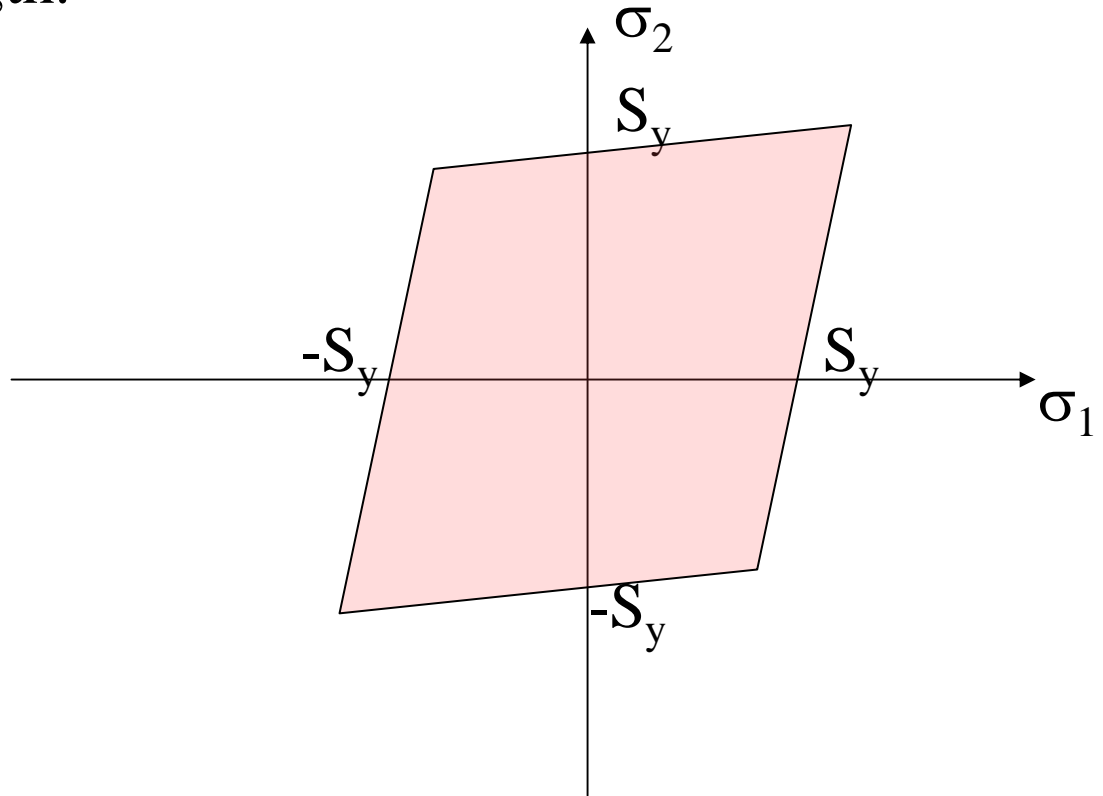
Also known as the Saint-Venant's Theory.

Applies only in the elastic range.

Failure is predicted to occur if $\sigma_1 - \nu\sigma_2 = \pm S_y$ or $\sigma_2 - \nu\sigma_1 = \pm S_y$

Where S_y is the yield strength.

For a biaxial state of stress



Maximum Strain-Energy Theory

Yielding is predicted to occur when the total strain energy in a given volume is greater than or exceeds the strain energy in the same volume corresponding to the yield strength in tension or compression.

The strain energy stored per unit volume (u_s) during uniaxial loading is

$$u_s = \frac{S_y^2}{2 \cdot E}$$

In a biaxial state of stress

$$u_\sigma = \frac{\varepsilon_1 \cdot \sigma_1}{2} + \frac{\varepsilon_2 \cdot \sigma_2}{2}$$

$$u_\sigma = \frac{1}{2E} (\sigma_1^2 + \sigma_2^2 - 2 \cdot \nu \cdot \sigma_1 \cdot \sigma_2)$$

This theory is no longer used

Comparison of Failure Theories to Experiments

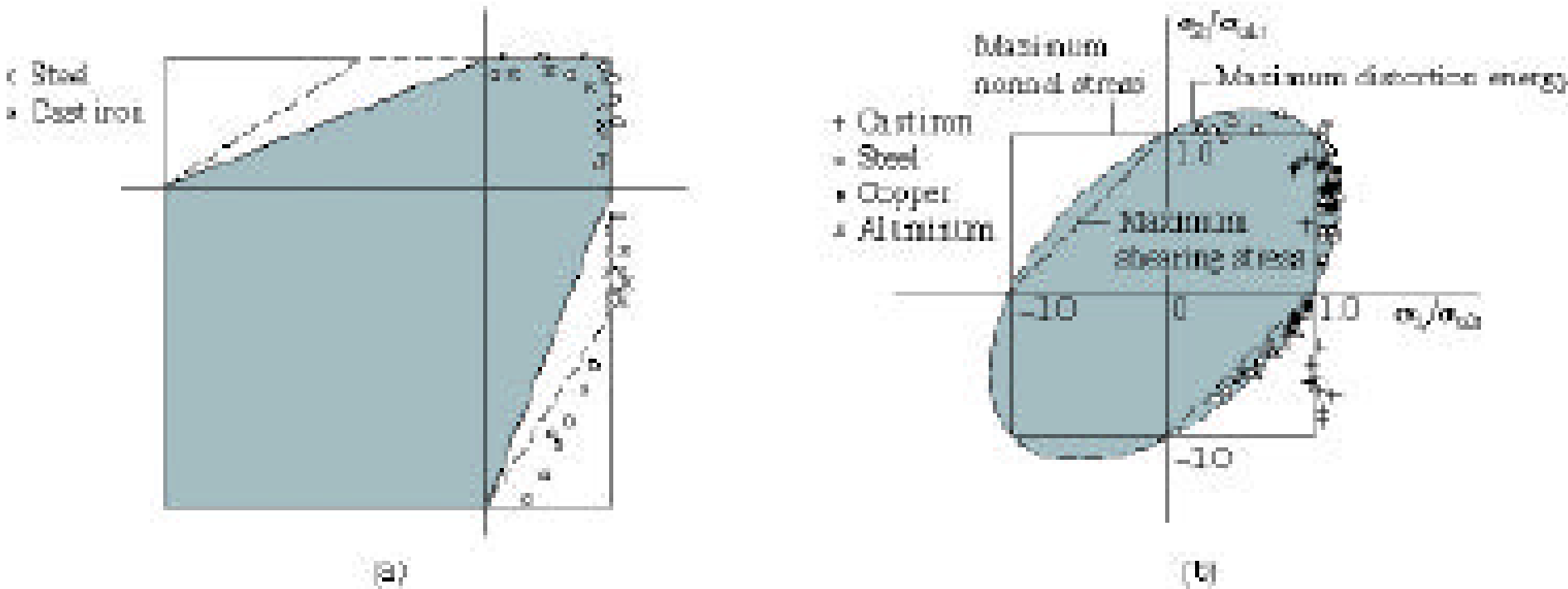


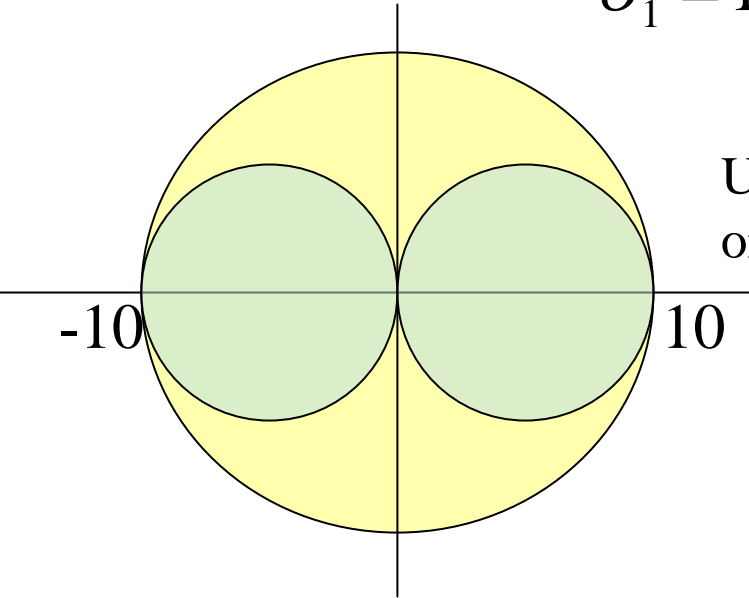
Figure 6.17: Comparison of experimental results to failure criterion. (a) Brittle fracture. (b) ductile yielding.

Example:

Given the material S_Y , σ_x , σ_y and τ_{xy} find the safety factors for all the applicable criteria.

a. Pure aluminum

$$S_Y = 30MPa \quad \sigma_x = 10MPa \quad \sigma_y = -10MPa \quad \tau_{xy} = 0MPa$$
$$\sigma_1 = 10MPa \quad \sigma_3 = -10MPa \quad \tau_{Max} = 10MPa$$



Is Al ductile or brittle? **Ductile**

Use either the Maximum Shear Stress Theory (MSST) or the Distortion Theory (DT)

MSST Theory

$$n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{30}{10 - (-10)} = \frac{30}{20} = 1.5$$

DT Theory $\sigma_{VM} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2} = \sqrt{300} = 17.32MPa$

$$n = \frac{S_y}{\sigma_{VM}} = \frac{30MPa}{17.32MPa} = 1.73$$

b. 0.2% C Carbon Steel

$$S_y = 65 \text{ Ksi} \quad \sigma_x = -5 \text{ Ksi} \quad \sigma_y = -35 \text{ Ksi} \quad \tau_{xy} = 10 \text{ Ksi}$$

In the plane XY the principal stresses are -1.973 Ksi and -38.03 Ksi with a maximum shear stress in the XY plane of 18.03 Ksi

In any orientation

$$\sigma_1 = 0 \text{ Ksi} \quad \sigma_2 = -1.973 \text{ Ksi} \quad \sigma_3 = -38.03 \text{ Ksi}$$

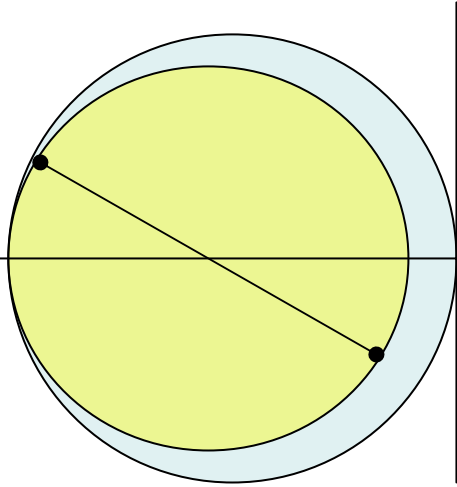
$$\tau_{Max} = 19.01 \text{ Ksi}$$

Ductile

MSST Theory $n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{65}{0 - (-38.03)} = 1.71$

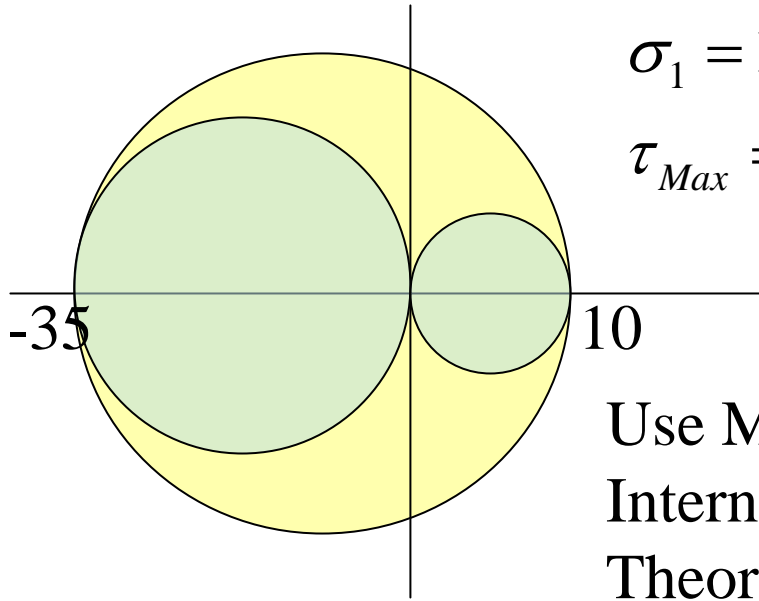
DT Theory $\sigma_{VM} = \sqrt{\sigma_1^2 + \sigma_3^2 - \sigma_1\sigma_3} = 38.03 \text{ Ksi}$

$$n = \frac{S_y}{\sigma_{VM}} = \frac{65 \text{ Ksi}}{38.03 \text{ MPa}} = 1.71$$



C. Gray Cast Iron

$$S_{ut} = 30\text{Ksi} \quad S_{uc} = 120\text{Ksi} \quad \sigma_x = -35\text{Ksi} \quad \sigma_y = 10\text{Ksi} \quad \tau_{xy} = 0\text{Ksi}$$



$$\sigma_1 = 10\text{Ksi} \quad \sigma_2 = 0\text{Ksi} \quad \sigma_3 = -35\text{Ksi}$$

$$\tau_{Max} = 22.5\text{Ksi}$$

Brittle

Use Maximum Normal Stress Theory (MNST),
Internal Friction Theory (IFT), Modified Mohr
Theory (MMT)

MNST Theory (tensile)

$$n = \frac{S_{ut}}{\sigma_1} = \frac{30}{10} = 3.0$$

MNST Theory (compression)

$$n = \frac{S_{uc}}{\sigma_3} = \frac{120}{35} = 3.4$$

IFT

$$\sigma_1 \geq 0 \quad \sigma_3 \leq 0 \quad 4^{\text{th}} \text{ _quadrant}$$

$$\text{line_equation} \quad \sigma_3 = -S_{uc} + \frac{S_{uc}}{S_{ut}} \sigma_1$$

$$\frac{1}{n} = \frac{\sigma_1}{S_{ut}} - \frac{\sigma_3}{S_{uc}} = \frac{10}{30} - \frac{-35}{120} = 0.625$$

$$n = 1.6$$

MMT

$$\sigma_1 \geq 0 \quad \sigma_3 \leq 0 \quad 4^{\text{th}} \text{ _quadrant}$$

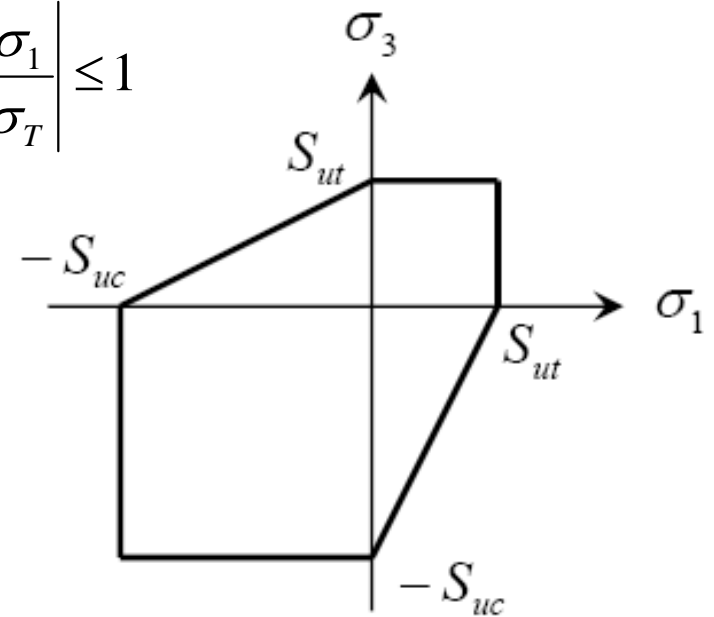
$$\sigma_1 - \frac{S_{ut}}{S_{uc} - S_{ut}} \sigma_3 = \frac{1}{n} \frac{S_{uc} S_{ut}}{S_{uc} - S_{ut}}$$

$$10 - \frac{30}{120 - 30} (-35) = \frac{1}{n} \frac{(120)(30)}{(120 - 30)}$$

$$\frac{1}{n} = 0.54$$

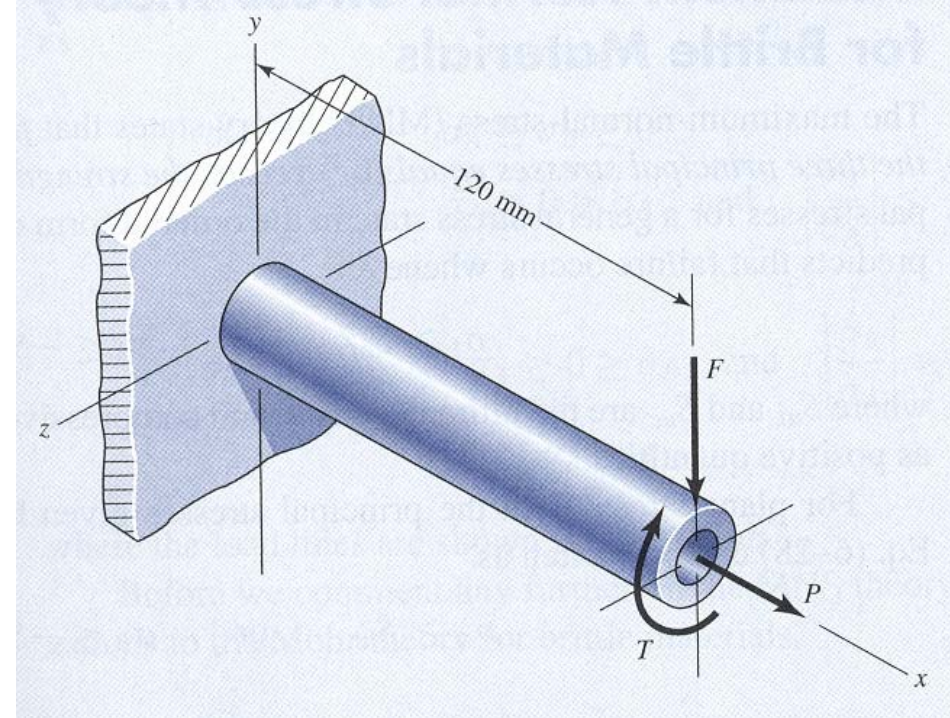
$$n = 1.84$$

$$\left| \frac{\sigma_2}{\sigma_C} - \frac{\sigma_1}{\sigma_T} \right| \leq 1$$



Example 1

The cantilever tube shown is to be made of 2014 aluminum alloy treated to obtain a specified minimum yield strength of 276MPa. We wish to select a stock size tube (according to the table below). Using a design factor of $n=4$.



The bending load is $F=1.75kN$, the axial tension is $P=9.0kN$ and the torsion is $T=72N.m$. What is the realized factor of safety?

Consider the critical area (top surface).

$$\sigma_x = \frac{P}{A} + \frac{Mc}{I}$$

Maximum bending moment = 120F

$$\sigma_x = \frac{9kN}{A} + \frac{120mm \times 1.75kNx \left(\frac{d}{2}\right)}{I}$$

$$\tau_{zx} = \frac{Tr}{J} = \frac{72 \times \left(\frac{d}{2}\right)}{J} = \frac{36d}{J}$$

$$\sigma_{VM} = \left(\sigma_x^2 + 3\tau_{zx}^2\right)^{1/2}$$

$$\sigma_{VM} \leq \frac{S_y}{n} = \frac{0.276}{4} GPa = 0.0690 GPa$$

For the dimensions of that tube

$$n = \frac{S_y}{\sigma_{VM}} = \frac{0.276}{0.06043} = 4.57$$

w_a = unit weight of aluminum tubing, lbf/ft
 w_s = unit weight of steel tubing, lbf/ft
 m = unit mass, kg/m
 A = area, in² (cm²)
 I = second moment of area, in⁴ (cm⁴)
 J = second polar moment of area, in⁴ (cm⁴)
 k = radius of gyration, in (cm)
 Z = section modulus, in³ (cm³)
 d, t = size (OD) and thickness, in (mm)

Size, in	w_a	w_s	A	I	k	Z	J
1 × 1/8	0.416	1.128	0.344	0.034	0.313	0.067	0.067
1 × 1/4	0.713	2.003	0.589	0.046	0.280	0.092	0.092
1 1/2 × 1/8	0.653	1.769	0.540	0.129	0.488	0.172	0.257
1 1/2 × 1/4	1.188	3.338	0.982	0.199	0.451	0.266	0.399
2 × 1/8	0.891	2.670	0.736	0.325	0.664	0.325	0.650
2 × 1/4	1.663	4.673	1.374	0.537	0.625	0.537	1.074
2 1/2 × 1/8	1.129	3.050	0.933	0.660	0.841	0.528	1.319
2 1/2 × 1/4	2.138	6.008	1.767	1.132	0.800	0.906	2.276
3 × 1/4	2.614	7.343	2.160	2.059	0.976	1.373	4.117
3 × 3/8	3.742	10.51	3.093	2.718	0.938	1.812	5.436
4 × 3/16	2.717	7.654	2.246	4.090	1.350	2.045	8.180
4 × 3/8	5.167	14.52	4.271	7.090	1.289	3.544	14.180

Size, mm	m	A	I	k	Z	J
12 × 2	0.490	0.628	0.082	0.361	0.136	0.163
16 × 2	0.687	0.879	0.220	0.500	0.275	0.440
16 × 3	0.956	1.225	0.273	0.472	0.341	0.545
20 × 4	1.569	2.010	0.684	0.583	0.684	1.367
25 × 4	2.060	2.638	1.508	0.756	1.206	3.015
25 × 5	2.452	3.140	1.669	0.729	1.336	3.338
30 × 4	2.550	3.266	2.827	0.930	1.885	5.652
30 × 5	3.065	3.925	3.192	0.901	2.128	6.381
42 × 4	3.727	4.773	8.717	1.351	4.151	17.430
42 × 5	4.536	5.809	10.130	1.320	4.825	20.255
50 × 4	4.512	5.778	15.409	1.632	6.164	30.810
50 × 5	5.517	7.065	18.118	1.601	7.247	36.226

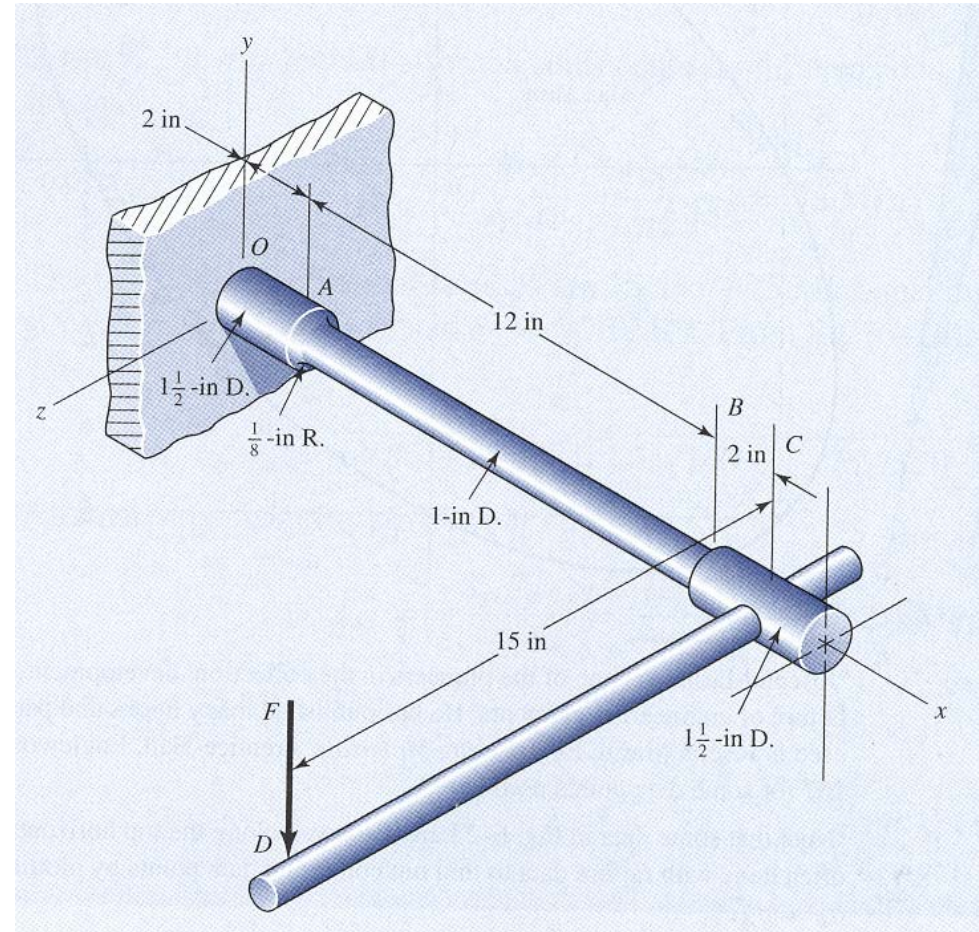
Example 2:

A certain force F is applied at D near the end of the **15-in** lever, which is similar to a socket wrench. The bar $OABC$ is made of **AISI 1035** steel, forged and heat treated so that it has a minimum (ASTM) yield strength of **81kpsi**. Find the force (F) required to initiate yielding. Assume that the lever DC will not yield and that there is no stress concentration at A .

Solution:

1) Find the critical section

The critical sections will be either point A or Point O . As the moment of inertia varies with r^4 then point A in the **1-in** diameter is the weakest section.



2) Determine the stresses at the critical section

$$\sigma_x = \frac{My}{I} = \frac{M \left(\frac{d}{2} \right)}{\frac{\pi d^4}{64}} = \frac{32 \times F \times 14 \text{in}}{\pi d^3} = 142.6F$$

3) Chose the failure criteria.

The AISI 1035 is a ductile material. Hence, we need to employ the distortion-energy theory.

$$\tau_{zx} = \frac{Tr}{J} = \frac{T \left(\frac{d}{2} \right)}{\frac{\pi d^4}{32}} = \frac{16 \times F \times 15 \text{in}}{\pi (1 \text{in})^3} = 76.4F$$

$$\sigma_{VM} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2} = \sqrt{\sigma_x^2 + 3\tau_{zx}^2} = 194.5F$$

$$F = \frac{S_y}{\sigma_{VM}} = \frac{81000}{194.5} = 416 \text{ lbf}$$

Apply the MSS theory. For a point undergoing plane stress with only one non-zero normal stress and one shear stress, the two non-zero principal stresses (σ_A and σ_B) will have opposite signs (Case 2).

$$\tau_{\max} = \frac{\sigma_A - \sigma_B}{2} = \frac{S_y}{2} = \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{zx}^2}$$

$$\sigma_A - \sigma_B \geq S_y = 2\sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{zx}^2} = \sqrt{\sigma_x^2 + 4\tau_{zx}^2}$$

$$81000 = \left((142.6F)^2 + 4 \times (76.4F)^2\right)^{1/2}$$

$$F = 388\text{ lbf}$$

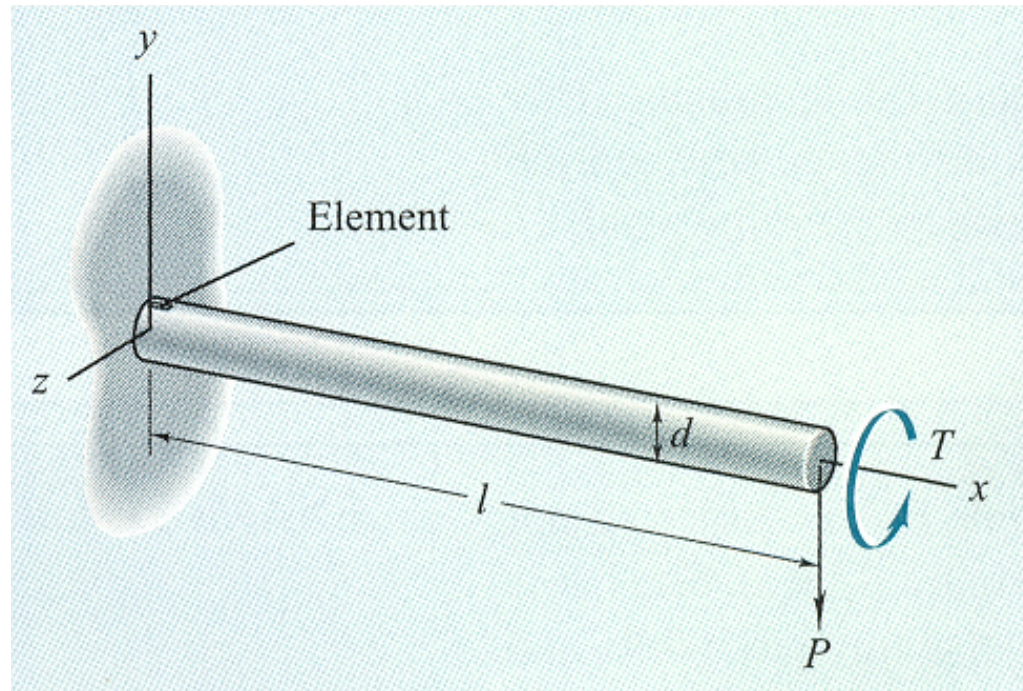
Example 3:

A round cantilever bar is subjected to torsion plus a transverse load at the free end. The bar is made of a ductile material having a yield strength of **50000psi**. The transverse force (**P**) is **500lb** and the torque is **1000lb-in** applied to the free end. The bar is **5in long (L)** and a **safety factor of 2** is assumed. Transverse shear can be neglected. Determine the minimum diameter to avoid yielding using both MSS and DET criteria.

Solution

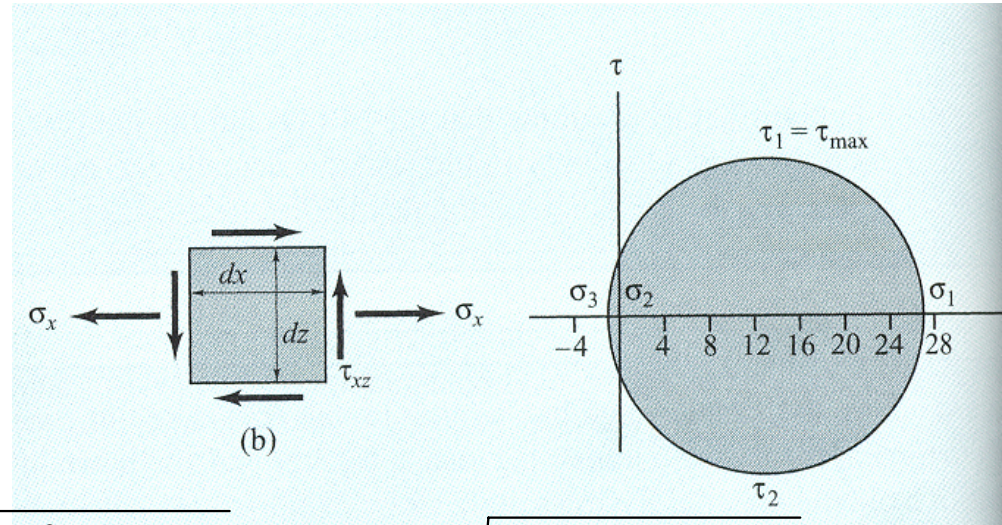
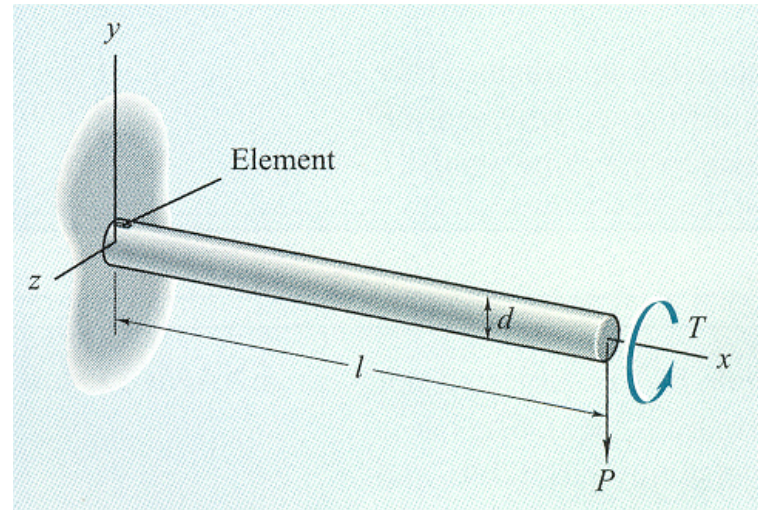
1) Determine the critical section

The critical section occurs at the wall.



$$\sigma_x = \frac{Mc}{I} = \frac{PL\left(\frac{d}{2}\right)}{\frac{\pi d^4}{64}} = \frac{32PL}{\pi d^3}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{T\left(\frac{d}{2}\right)}{\frac{\pi d^4}{32}} = \frac{16T}{\pi d^3}$$



$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} = \left(\frac{\sigma_x}{2}\right) \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2}$$

$$\sigma_{1,2} = \frac{16PL}{\pi d^3} \pm \sqrt{\left(\frac{16PL}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3} \left[PL \pm \sqrt{(PL)^2 + T^2} \right]$$

$$\sigma_{1,2} = \frac{16}{\pi d^3} \left[500 \times 5 \pm \sqrt{(500 \times 5)^2 + 1000^2} \right]$$

$$\sigma_1 = \frac{26450}{d^3}$$

$$\sigma_2 = -\frac{980.8}{d^3}$$

The stresses are in the wrong order.. Rearranged to

$$\sigma_1 = \frac{26450}{d^3}$$

$$\sigma_3 = -\frac{980.8}{d^3}$$

MSS

$$\tau_{MAX} = \frac{\sigma_1 - \sigma_3}{2} = \frac{26450 - (-980.8)}{2d^3} = \frac{13715.4}{d^3}$$

$$\sigma_1 - \sigma_3 = 2\tau_{MAX} \leq \frac{S_y}{n} = \frac{50000}{2} = 25,000$$

$$d \geq 1.031 \text{ in}$$

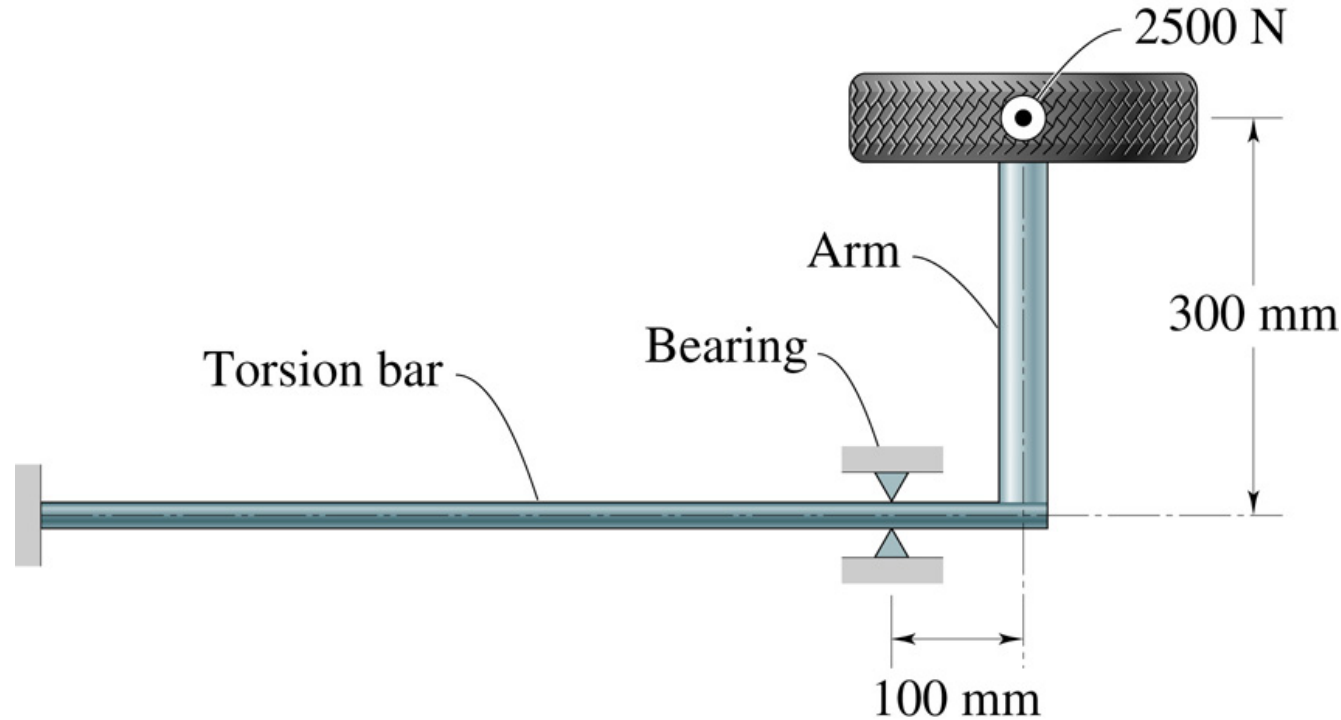
DET

$$\sigma_{VM} = \sqrt{\sigma_1^2 + \sigma_3^2 - \sigma_1\sigma_3} = \sqrt{\left(\frac{26450}{d^3}\right)^2 + \left(-\frac{980.8}{d^3}\right)^2 - \left(\frac{26450}{d^3}\right)\left(-\frac{980.8}{d^3}\right)}$$

$$\sigma_{VM} = \frac{26950}{d^3} \leq \frac{S_y}{n} = \frac{50000}{2}$$

$$d \geq 1.025 \text{ in}$$

Example 4:



In the wheel suspension of a car, the spring motion is provided by a torsion bar fastened to the arm on which the wheel is mounted. The torque in the torsion bar is created by the 2500N force acting on the wheel from the ground through a 300mm long lever arm. Because of space limitations, the bearing holding the torsion bar is situated 100mm from the wheel shaft. The diameter of the torsion bar is 28mm. Find the stresses in the torsion bar at the bearing by using the DET theory.

Solution

The stresses acting on a torsion bar are:

1. Torsion
$$\tau = \frac{Tc}{J} = \frac{(F \times arm_length)\left(\frac{d}{2}\right)}{\frac{\pi d^4}{32}} = 32 \frac{(2500 \times 0.3)(0.014)}{\pi(0.028)^4} Pa = 174MPa$$

2. bending
$$\sigma = \frac{Mc}{I} = \frac{(F \times bearing_length)\left(\frac{d}{2}\right)}{\frac{\pi d^4}{64}} = 64 \frac{(2500 \times 0.1)(0.014)}{\pi(0.028)^4} Pa = 116MPa$$

The principal stresses are:

$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} = \left(\frac{\sigma_x}{2}\right) \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2}$$

$$\sigma_{1,2} = \frac{116.0}{2} \pm \sqrt{\left(\frac{116.0}{2}\right)^2 + (174.0)^2}$$

$$\sigma_1 = 241.4MPa$$

$$\sigma_2 = -125.4MPa$$

DET

$$\sigma_{VM} = \sqrt{\sigma_1^2 + \sigma_3^2 - \sigma_1\sigma_3} = \sqrt{(241.4)^2 + (-125.4)^2 - (241.4)(-125.4)}$$

$$\sigma_{VM} = 322.6MPa \leq \frac{S_y}{n}$$

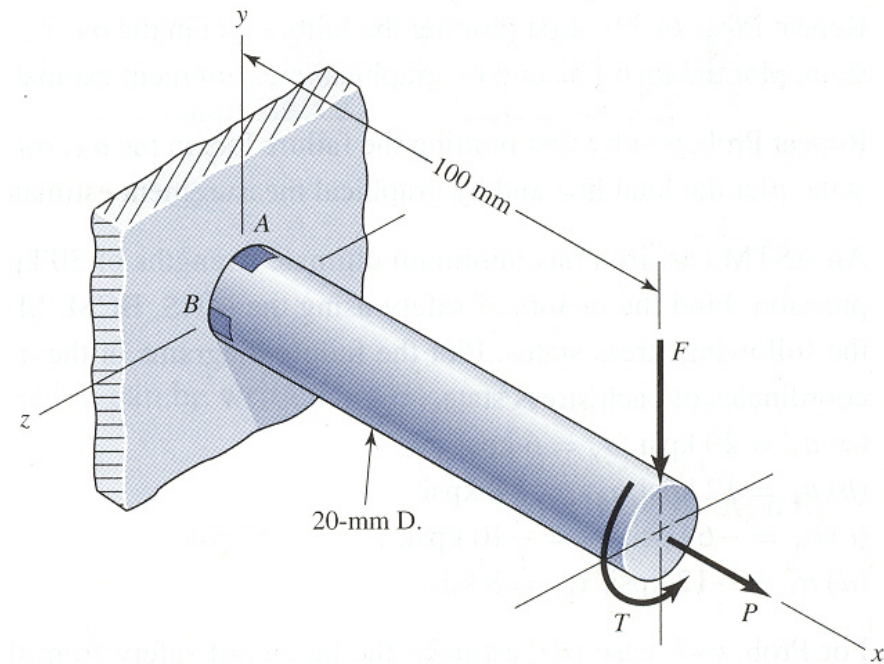
MSS

$$\tau_{Max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{241.4 - (-125.4)}{2} = 183.4MPa$$

$$2 \times \tau_{Max} = 366.8MPa \leq \frac{S_Y}{n}$$

Example 5:

The factor of safety for a machine element depends on the particular point selected for the analysis. Based upon the DET theory, determine the safety factor for points A and B.



This bar is made of AISI 1006 cold-drawn steel ($S_y=280\text{MPa}$) and it is loaded by the forces $F=0.55\text{kN}$, $P=8.0\text{kN}$ and $T=30\text{N.m}$

Solution:

Point A

$$\sigma_x = \frac{Mc}{I} + \frac{P}{Area} = \frac{Fl\left(\frac{d}{2}\right)}{\frac{\pi d^4}{64}} + \frac{P}{\frac{\pi d^2}{4}} = \frac{32Fl}{\pi d^3} + \frac{4P}{\pi d^2}$$

$$\sigma_x = \frac{32(0.55)(10^3)(0.1)}{\pi(0.02)^3} + \frac{4(8)(10^3)}{\pi(0.02)^2} = 95.49\text{MPa}$$

$$\tau_{xy} = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(30)}{\pi(0.020)^3} = 19.10 \text{ MPa}$$

$$\sigma_{VM} = \sqrt{(\sigma_x^2 + 3\tau_{xy}^2)} = \left[95.49^2 + 3(19.1)^2\right]^{1/2} = 101.1 \text{ MPa}$$

$$n = \frac{S_y}{\sigma_{VM}} = \frac{280}{101.1} = 2.77$$

Point B

$$\sigma_x = \frac{4P}{\pi d^2} = \frac{4(8)(10^3)}{\pi(0.02)^2} = 25.47 \text{ MPa}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} + \frac{4V}{3A} = \frac{16(30)}{\pi(0.02)^3} + \frac{4(0.55)(10^3)}{3\left(\frac{\pi}{4}\right)(0.02)^2} = 21.43 \text{ MPa}$$

$$\sigma_{VM} = \left[25.47^2 + 3(21.43)^2\right]^{1/2} = 45.02 \text{ MPa}$$

$$n = \frac{280}{45.02} = 6.22$$